Observational Constraints on Accelerating Brane Cosmology with Exchange between the Bulk and Brane

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(Dated: February 5, 2008)

We explore observational constraints on a cosmological brane-world scenario in which the bulk is not empty. Rather, exchange of mass-energy between the bulk and the bane is allowed. The evolution of matter fields to an observer on the brane is then modified due to new terms in the energy momentum tensor describing this exchange. We investigate the constraints from various cosmological observations on the flow of matter from the bulk into the brane. Interestingly, we show that it is possible to have a $\Lambda = 0$ cosmology to an observer in the brane which satisfies standard cosmological constraints including the CMB temperature fluctuations, Type Ia supernovae at high redshift, and the matter power spectrum. This model even accounts for the observed suppression of the CMB power spectrum at low multipoles. In this cosmology, the observed cosmic acceleration is attributable to the flow of matter from the bulk to the brane. A peculiar aspect of this cosmology is that the present dark-matter content of the universe may be significantly larger than that of a ΛCDM cosmology. Its influence, however, is offset by the dark-radiation term. Possible additional observational tests of this new cosmological paradigm are suggested.

PACS numbers: 98.80.Cq, 98.65.Dx, 98.70.Vc

I. INTRODUCTION

A puzzling question in modern cosmology has been the nature and origin of the dark energy responsible for the apparent acceleration [1] of the universe in the present epoch. The simplest explanation is, perhaps, that of a cosmological constant, or a vacuum energy in the form of a "quintessence" scalar field slowly evolving along an effective potential. The existence of a cosmological constant or quintessence, however, leads inevitably to the well known fine tuning problem as to why the present dark energy is so small compared to the natural scales of high-energy physics. There is also a cosmic coincidence problem as to why the universe has conspired to produce nearly equivalent energy contents in matter and dark energy at the present time. Moreover, most quintessence models are now ruled out [2] by the fact that the equation of state parameter, $w \equiv p/\rho$ is so close to -1. This implies that there is little evidence for evolution in a quintessence field so that a cosmological constant remains the most likely interpretation.

Accounting for such dark energy, however, poses an unsolved theoretical challenge. Ultimately, one would hope that the existence of this dark energy could be accounted for in the context of string theory or some other unified theory. This problem is exacerbated, however, by the fact that it appears difficult to accommodate a cosmological constant in string theory [3], although some progress has been made [4]. Recently, it has been proposed [5, 6] that the observed cosmic acceleration may be an artifact of inhomogeneities in the distribution of mass-energy. That possibility, though compelling for its simplicity, has not yet been established [7, 8].

Hence, in this paper, we consider it worthwhile to analyze an alternative mechanism by which the observed cosmic acceleration could be produced even without the need to invoke dark energy and its associated complexities. Specifically, we explore models in which the cosmic acceleration is driven [9, 10, 11, 12, 13, 14] by the flow of dark matter from a higher dimension (the bulk) into our three-space (the brane).

This study is also in part motivated by the currently popular view that our universe could be a sub-manifold embedded in a higher-dimensional space-time. This paradigm derives from the low-energy limit of the heterotic M-theory [15] which becomes an 11-dimensional supergravity compactified on a line segment to two 10-dimensional $E8 \times E8$ gauge theories. Hence, the universe can be envisioned as two smooth 10-dimensional manifolds (9-branes) embedded in a bulk dimension.

A thin three-brane (Randall-Sundrum brane) embedded in a five-dimensional anti-de Sitter space AdS_5 has been proposed as a practical phenomenological model [16] in which to explore such higher dimensional physics. This approach poses an alternative to the standard Kaluza-Klein (KK) compactification of extra dimensions, through the localization of the graviton zero mode on the brane. This brane approach also provides a new way of understanding the hierarchy between the four-dimensional Planck scale $M_{\rm pl}$ and the electro-weak scale

[17, 18].

Of relevance to the present study is that in this phenomenological approach, one can relax the requirement that the gauge groups be rigorously confined to the brane. For example, particles might be localized on a defect in the higher dimensional space [19, 20]. The simplest example of this would be a domain wall (brane) in (4+1) dimensions. In such a picture, the extra (bulk) dimension can be infinite and the observed 3-dimensional fields are represented by zero modes of bulk fields in the domain-wall background. These modes are localized and thus behave like 3-dimensional mass-less fields. In this domain-wall scenario, physical matter fields are dynamically confined to this sub-manifold, while gravity can reside in the extra bulk dimension.

The stability of massive matter fields has been analyzed [21] in this scenario where it was shown that such massive particles are metastable on the brane. That is, for both scalar and fermion fields, the quasi-normal modes are metastable states that decay into continuum states. From the viewpoint of an observer in the 4-dimensional space time, these massive particles appear to propagate for some time in three spatial dimensions and then disappear into the fifth dimension. This disappearance of massive particles from the 3-brane constitutes an energy flow from the brane to the bulk. The cosmological constraints on such disappearing matter have been studied in Ref. [22].

Moreover, if the massive particles can also exist in the bulk, it becomes possible to consider the inverse flow from the bulk into our three-brane. In this paper, we build a model with such mass-energy exchange, in which the flow from the bulk to the brane provides the present observed acceleration [9, 10, 11, 12, 13, 14] of the universe. As noted in Ref. [22] a heavy (∼TeV) dark-matter candidate such as the lightest supersymmetric partner is likely to have the largest tunneling rate between the brane and bulk. Moreover, a recent study [23] of energy transfer from the bulk through parametric resonance indicates that massive particles are most efficient for depositing energy on the brane. Hence, in the present work we mainly consider the exchange between the bulk and brane involving a growth of the cold dark-matter component on the brane. This model [which we refer to as growing cold dark matter (GCDM)] is, thus, an alternative to the standard Λ plus cold dark matter (SΛCDM) cosmology to an observer on the 3-brane.

We test this model by analyzing the observations of Type Ia supernovae at high redshift, the temperature fluctuation spectrum of the cosmic microwave background (CMB), and the matter power spectrum. Surprisingly, all of these constraints can be satisfied in this model even without introducing a cosmological constant on the brane. A shocking feature of the best fit model, however, is that the present dark-matter content becomes as much as an order of magnitude larger than that of a $S\Lambda$CDM cosmology. The influence of this excess dark matter, however, is suppressed by the large dark-radiation component associated with the flow of matter from the bulk dimension. In fact, this model provides a natural explanation for the observed suppression of the CMB power spectrum for the lowest multipoles due to the effect of the late arriving dark matter on the integrated Sachs-Wolf effect. We show here that the matter power spectrum exhibits the correct power on large and small scales, however, a somewhat large bias factor is required.

II. MODEL WITH ENERGY EXCHANGE BETWEEN THE BULK AND BRANE

We begin with the five-dimensional Einstein equations

$$G_{AB} = \kappa^2 T_{AB}, \tag{1}$$

where $\kappa^2 = M^{-3}$ is the 5-dimensional gravitation constant with M the 5-dimensional Planck mass.

T_{AB} is the five-dimensional total energy-momentum tensor. We use the index notation $A, B = (0, 1, 2, 3, 5)$ to clearly distinguish the bulk fifth dimension. The metric is assumed to be of the Randall-Sundrum (RS) [16] form. For simplicity, however, we consider a coordinate system with a static bulk dimension and an expanding brane or moving [24] brane. Hence, we choose a metric in Gauss-normal coordinates

$$ds^2 = -\hat{n}^2(t,y)dt^2 + \hat{a}^2(t,y)\gamma_{ij}dx^i dx^j + dy^2 \quad , \tag{2}$$

where γ_{ij} is a maximally symmetric 3-metric. As an illustrative model, we allow the Hubble expansion of the brane relative to a static bulk to produce a nonzero 5-component of the bulk fluid velocity as discussed below.

A. Energy-Momentum Tensor

When matter is present in the bulk, the evolution of the brane is not autonomous. It becomes necessary to explicitly define the bulk energy-momentum tensor. The general bulk energy-momentum tensor for a perfect fluid in this 5-dimensional space-time is,

$$T^A_{\ B} = (\tilde{\rho} + \tilde{p})U^A U_B + \delta^A_{\ B}\tilde{p} \quad , \tag{3}$$

where $\tilde{\rho}$ and $\tilde{p}$ denote density and pressure in 4-dimensional space. In Gaussian-normal coordinates, however, the bulk 5th dimension is fixed so that the four density and pressure are proportional to the three-volume like ordinary three-space quantities. For normal matter on the brane we denote the density and pressure as ρ and p. In our model in which dark matter can exist both on the brane and in the bulk we denote the dark-matter three density and pressure on the brane as $\bar{\rho}$ and $\bar{p}$. Since the density, pressure and equation of state can be different in the bulk we denote the four density of dark matter in the bulk as $\hat{\rho}$ and $\hat{p}$.

In a frame for which the three-brane is at rest we can decompose the energy-momentum tensor into three parts.

$$T_{AB} = \delta(y)\left(T_{AB}^{BRANE}\right) + T_{AB}^{BULK} + T_{AB}^{DM} \quad , \qquad (4)$$

where the δ-function identifies the location of the brane at $y = 0$ to which the standard-model particles are assumed to be confined. ${}^{(BRANE)}T^A_B$, corresponds to the usual three-density and pressure, ρ and p, of ordinary relativistic and non-relativistic particles plus the tension τ on the brane.

$${}^{(BRANE)}T^A_B = \mathrm{diag}(-\tau - \rho, -\tau + p, -\tau + p, -\tau + p, 0) \quad . \qquad (5)$$

The bulk is taken to be AdS_5, so that the five-dimensional cosmological constant Λ_5 is negative in the bulk. Hence, we write the vacuum energy-momentum for the bulk, ${}^{(BULK)}T^A_B$, as

$${}^{(BULK)}T^A_B = \mathrm{diag}(-\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5, -\Lambda_5). \qquad (6)$$

In our model dark matter is allowed to exist both on the brane and in the bulk. Hence, we decompose the dark-matter energy-momentum tensor, ${}^{(DM)}T^A_B$, into the usual three-density $\bar{\rho}$ and pressure $\bar{p}$ of dark matter on the brane, plus the bulk components ${}^{(DM-BULK)}T^A_B$.

$${}^{(DM)}T^A_B = \delta(y)\mathrm{diag}(-\bar{\rho}, \bar{p}, \bar{p}, \bar{p}, 0) + {}^{(DM-BULK)}T^A_B. \qquad (7)$$

where,

$${}^{(DM-BULK)}T^0_5 \sim (\hat{\rho} + \hat{p})U_5 \quad , \qquad (8)$$

represents the matter-energy flow from the bulk to the brane, while

$${}^{(DM-BULK)}T^5_5 = (\hat{\rho} + \hat{p})U^5 U_5 + \hat{p} \quad , \qquad (9)$$

represents a bulk pressure in the limit of vanishing U_5.

As we shall see, the cosmology for an observer living on the three-brane will depend upon the equation of state (EOS) properties for matter in the bulk dimension. It is not guaranteed, however, that matter will have the same EOS properties in the bulk as on the brane. Inevitably, assumptions must be made [10, 25] about the form of the energy-momentum tensor for matter in the bulk. For example, a bulk field [26], or radiation emitted from the brane to the bulk [27] have been considered along with possible thermal effects [28]. In a string theory sense, particles in the bulk may appear as a topological defect between two branes. For the purposes of this paper, we adopt [9, 10] the usual parametrization of the EOS for matter in the bulk. Scaling the bulk dark-matter density to the present critical density ρ_{cr} on the brane, we write:

$$\hat{\rho} \propto \left(\frac{\rho_{cr}}{a^q}\right) \quad , \qquad (10)$$

where $a(t)$ is the scale factor on the brane, defined as $a(t) = \hat{a}(t, 0)$, and for a fixed bulk dimension the scaling parameter q can be written $q = 3(1 + \hat{w})$, where $\hat{w} = \hat{p}/\hat{\rho}$ is the usual equation of state parameter with, $\hat{w} = 0$ for normal cold dark matter, while $\hat{w} = 1/3$ for relativistic matter. A value of $\hat{w} = -1$ corresponds to a vacuum energy, while $\hat{w} = -2/3$ for a string-like topological defect.

To illustrate this cosmology we consider two models for the motion of matter in the bulk relative to the brane. In the Gaussian-normal coordinates, the five velocity of matter in the static bulk with respect to the expanding 3-brane is just [29]

$$U_5 \propto -lH \quad , \qquad (11)$$

where

$$l = [-6M^3/\Lambda_5]^{1/2} \quad , \qquad (12)$$

is the bulk curvature radius [22].

As a second illustration we also consider the case of a constant U_5 component. Such a model might correspond, for example, to gravitational accretion of matter from the bulk toward the brane at a constant rate.

For the purposes of scaling, we parameterize the 0-5 component of the bulk dark-matter energy-momentum tensor as

$${}^{(DM-BULK)}T^0_5 = -\frac{\alpha}{2}\left(\frac{\rho_{cr}}{a^q}\right) \times H \quad , \qquad (13)$$

where we have assumed a constant transition rate of matter between the bulk and the brane absorbed into the dimensionless parameter α which can be either positive or negative. For the case of constant U_5 we replace $H \to H_0$ in the above scaling, where H_0 is the present Hubble parameter.

B. Accelerating Cosmologies

The cosmological equations of motion with brane-bulk energy exchange have previously been formulated in Refs. [9, 10, 11, 12, 13, 14]. The covariant derivative of the energy-momentum tensor leads to the usual energy conservation condition for various components i of normal matter on the brane;

$$\frac{\dot{\rho}_i}{\rho_i} + 3(1 + w_i)\,\frac{\dot{a}}{a} = 0 \quad , \qquad (14)$$

plus a new condition for the dark matter which takes into account the flow of matter from the brane world,

$$\frac{\dot{\bar{\rho}}}{\bar{\rho}} + 3(1 + \bar{w})\frac{\dot{a}}{a} = -\frac{T}{\bar{\rho}}, \qquad (15)$$

where $T \equiv 2T^0_5$ is the discontinuity of the (0,5) component of ${}^{(DM-BULK)}T^A_B$ at $y = 0$. The (0,0) component of the Einstein equation produces a modified Friedmann

cosmology to an observer on the brane:

$$H^2 = \frac{\dot a^2}{a^2} \;=\; \frac{8}{3}\pi G_N(\rho+\bar\rho) - \frac{k}{a^2} + \Lambda_4 + \frac{\kappa^4}{36}(\rho+\bar\rho)^2 + \chi \quad , \qquad (16)$$

while the (5,5) component leads to an equation for the dark radiation term χ,

$$\dot\chi + 4\,\frac{\dot a}{a}\,(\chi + \frac{\kappa^2}{6}T^5_5) = \frac{\kappa^4}{18}\,(\rho+\bar\rho+\tau)\,T \quad . \quad (17)$$

In Eqs. (16) and (17) the usual change variables has been introduced:

$$^{(DM-BULK)}T^5_{\;5} \equiv T^5_{\;5} \quad , \qquad (18)$$

$$\Lambda_4 \equiv (\kappa^2/6)\left[\Lambda_5 + (\kappa^2/6)\tau^2\right] \quad , \qquad (19)$$

and

$$\kappa^4\tau/18 \equiv 8\pi G_N/3 \quad . \qquad (20)$$

Introducing these identities into Eq. (16) leads to the usual Friedmann equation plus extra terms with χ and $(\rho+\bar\rho)^2$. This modified Friedmann equation can be normalized in the usual way so that the spatial curvature takes on values $k = -1, 0, 1$. In the limit of an empty bulk and no exchange between the bulk and the brane, $T^0_5 = T^5_5 = 0$, the quantity χ varies with scale factor on the brane as C/a^4. Thus, χ reduces to the standard dark-radiation term when all matter fields are confined to the brane.

Now we assume that the dark matter is cold on the brane: $\bar p = 0$. First, we consider the special case $T = \Gamma\bar\rho$. A solution of Eq. (15) is $\bar\rho = C\exp(-\Gamma t)/a^3$. This is precisely the disappearing dark matter cosmology introduced in our previous paper [22]. Thus, $T = \Gamma\bar\rho$ describes matter-energy flow from the brane into the bulk. In this cosmology dark-matter particles can exist in both the brane and the bulk. Hence, negative T describes a reverse flow from the bulk to the brane. However, in the present paper, we only consider the inflow from the bulk into the brane.

1. *An Illustration*

To see how an accelerating cosmology arises, consider the simple example of a constant flow rate from the bulk into the brane, i.e. $T = -\Sigma$ for a positive constant Σ. In the case of $k = 0$ and $\rho << \bar\rho$, (nearly realized in the present universe) equations (15)-(17) have a fixed point $(H_*, \bar\rho_*, \chi_*)$ governed by,

$$3\bar\rho_* H_* = \Sigma, \qquad (21)$$

$$H_*^2 = \frac{8}{3}\pi G_N\bar\rho_* + \Lambda_4 + \frac{\kappa^4}{36}{\bar\rho_*}^2 + \chi_*, \qquad (22)$$

$$4H_*(\chi_* + \frac{\kappa}{6}T^5_5) = -\frac{\kappa^4}{9}\tau\Sigma \quad . \qquad (23)$$

The constant r.h.s. of equation (22) then is manifestly equivalent to a cosmological-constant dominated universe.

In general, the fixed point remains even in the case of $\Lambda_4 = 0$. Thus, the present accelerating universe need not be attributed to Λ_4 at all, but could also arise from constant bulk components of the five dimensional stress-energy tensor, $\Sigma = -2T^0_5$ and T^5_5. When a fluid is static, T^5_5 represents a pressure, it is then natural to set the pressure to be zero. But T^5_5 is in general non-vanishing for a moving flow. Another possibility for non-vanishing T^5_5 is that the dark matter in the bulk is in a vacuum state. In this case, T^5_5 is a vacuum pressure, and it is likely that $\Sigma = 0$.

C. $\Lambda_4 = 0$ Accelerating cosmology

We consider two growing dark matter models of interest to the present work. For the case of vanishing Λ_4 and constant U_5, the evolution equations of energy densities are rewritten as,

$$\dot{\bar\rho} + 3H\bar\rho = \alpha/a^q \times \rho_{cr}H_0, \qquad (24)$$

$$\dot\rho + 3H(\rho+p) = 0\ , \qquad (25)$$

$$\dot\rho_\chi + 4H\rho_\chi = -\alpha/a^q \times \rho_{cr}H_0, \qquad (26)$$

where $\rho_\chi = \chi/(8\pi G_N/3)$ is the dark-radiation term which plays the role of dark energy. For the case of $U_5 = -lH$, the evolution equations for the energy densities are

$$\dot{\bar\rho} + H\left[3\bar\rho - \alpha/a^q \times \rho_{cr}\right] = 0\ , \qquad (27)$$

$$\dot\rho + 3H(\rho+p) = 0\ , \qquad (28)$$

$$\dot\rho_\chi + H\left[4\rho_\chi + \alpha/a^q \times \rho_{cr}\right] = 0\ . \qquad (29)$$

These equations satisfy the conservation's law on the brane. Note, that in this latter case, the fact that $\rho_\chi < 0$ means that Eq. (29) quickly evolves to $\dot\rho_\chi = 0$ for the case when $q = 0$. Indeed, as long as $q < 3$ an accelerating cosmology eventually emerges [10]. Hence, the dark radiation contribution becomes constant and indistinguishable from a cosmological constant.

For the $\Lambda_4 \equiv \Lambda = 0$ accelerating cosmology of interest here, the modified Friedmann equation for cosmic expansion can be written

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}(\bar\rho + \rho + \rho_\chi) + \frac{k}{a^2}\ . \qquad (30)$$

Here we have neglected the ρ^2 term. This term decays rapidly as a^{-8} in the early radiation dominated epoch and hence is insignificant for the present studies. Note, that Eq. (30) corresponds to the limit in which $T_5^5 \approx 0$, and $\tau >> MAX(\rho, \bar{\rho})$.

Figure 1 illustrates the evolution of various components on the brane in a simple $\Lambda = k = 0$ cosmology. This cosmology separates into three characteristic epochs on this figure. First, the usual early radiation dominated epoch ($a < 10^{-4}$). Second, a dark-matter dominated epoch ($10^{-4} < a < 10^{-1}$). Third, the dark matter and dark radiation dominated accelerating epoch ($a > 0.5$).

Both of our two models for the growth rate of dark matter are illustrated on Figure 1 for the case of $q = 0$, $\alpha = 8$. During the first and second epochs, the dark radiation component for the case of constant U_5 evolves as $\rho_{DR} \propto a^{2-q}$ and $\rho_{DR} \propto a^{3/2-q}$, respectively. For the case of brane expansion into the bulk, $U_5 \propto H$, the dark radiation term is initially much larger than for the case of constant U_5 and it remains at a nearly constant value. Nevertheless, during the radiation- and matter-dominated epochs, the dark radiation term for both models is insignificant until recent history near $z \approx 1$. Hence, this cosmology is not constrained by primordial nucleosynthesis. [30]

This model thus explains the smallness problem of the apparent cosmological constant as simply due to the slow tunneling of matter from the bulk onto the brane. The cosmic acceleration only occurs now, because this is the epoch for which the fixed-point solution could be obtained. Moreover, in what follows we show that the fits to observations based upon these two models are indistinguishable from each other and from the best standard $S\Lambda$CDM model.

Eventually, in the third region the cosmology of interest to this paper emerges. The dark radiation component dominates. In the case of $q \approx 0$ it becomes a constant dark-energy density. Hence, the dark radiation associated with in-flowing cold dark matter leads the cosmic acceleration without the need for a cosmological constant on the right hand side of Eq. (30) as long as matter in the bulk has the right equation of state.

III. OBSERVATIONAL CONSTRAINTS

Having defined the cosmology of interest we now analyze the various cosmological constraints as a test of this hypothesis by solving equations (24)-(30) numerically. In particular, we examine the magnitude-redshift relation for type Ia supernovae (SNIa), the cosmic microwave background (CMB) and the matter power spectrum $P(k)$. Parameters summarizing the best fits to these constraints are given in Table 1 for models with and without growing cold dark matter. The SΛCDM fits are consistent with the usually inferred cosmological parameters (e.g. [2]. An important constraint on the GCDM model is that the sum of $\Omega_{DM} + \Omega_{DR}$ be approximately

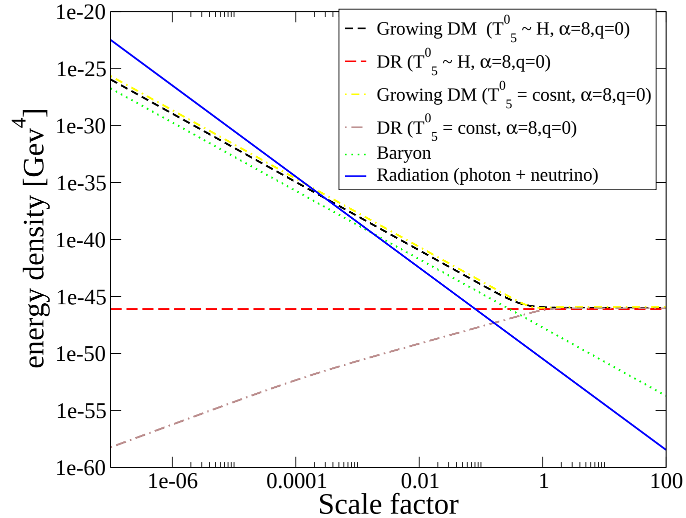

FIG. 1: Evolution of various energy densities as a function of scale factor in a $\Lambda = k = 0$ growing cold-dark-matter model. The curves labeled as "Growing DM" shows the dark radiation component. Note that the dark radiation content is plotted as its absolute value. This quantity is actually negative in these models.

equivalent to the sum of $\Omega_{DM} + \Omega_\Lambda$ in the standard cosmology. This is evidenced by the fifth column of Table 1. Note, however, that the magnitude of Ω_{DM} and Ω_{DR} individually can be quite large in the GCDM model, as long as $\Omega_{tot} \approx 1$.

IV. SUPERNOVA LIKELIHOOD ANALYSIS

The apparent brightness of the Type Ia supernova standard candle with redshift is given [31] by a simple relation which we slightly modify to incorporate the brane-world cosmology given in Eq. (30). The luminosity distance becomes,

$$\begin{aligned} D_L &= \frac{c(1+z)}{H_0\sqrt{\Omega_{k0}}} sinn\Bigg\{\sqrt{\Omega_{k0}}\int_0^z dz'[\Omega_\gamma(z') \\ &\quad +(\Omega_{DM}(z') + \Omega_B(z')) \\ &\quad +\Omega_{k0}(1+z')^2 + \Omega_\Lambda + \Omega_\chi(z')\Big]^{-1/2}\Bigg\} \ , \quad (31) \end{aligned}$$

where H_0 is the present value of the Hubble constant, and $sinn(x) = \sinh x$ for $\Omega_{k0} > 0$, $sinn(x) = x$, for $\Omega_{k0} = 0$ and $sinn(x) = \sin x$ for $\Omega_{k0} < 0$. The Ω_i are the energy densities normalized by the current critical density, i.e. $\Omega_i(z) = 8\pi G\rho_i(z)/3H_0^2$. Note the Ω_{DM} has a nontrivial redshift dependence in the present cosmology via Eq. (15).

The look-back time $t_0 - t$ is a function of redshift be-

comes,

$$
\begin{aligned}
t_0 - t \;=\; & H_0^{-1}\left\{ \int_0^z (1+z')^{-1} \Big[\Omega_\gamma(z') \right. \\
& + \; \Omega_B(z') + \Omega_{DM}(z') \\
& \left. + \; \Omega_{k0}(1+z')^2 + \Omega_\Lambda + \Omega_\chi(z') \Big]^{-1/2} dz' \right\} \quad (32)
\end{aligned}
$$

We have found the best fits to the supernova magnitude-redshift relation by maximizing the likelihood functions in an effective χ^2 analysis,

$$
\begin{aligned}
-2\ln\mathcal{L}_{SN} \;&=\; \chi^2_{\rm eff} \\
&= \sum \frac{(Y_i^{data} - Y_i^{calc})^2}{\sigma_i^2} \\
&- \left(\sum \frac{Y_i^{data} - Y_i^{calc}}{\sigma_i^2} \right)^2 \Big/ \sum \frac{1}{\sigma_i^2}. \quad (33)
\end{aligned}
$$

where the second term corresponds to and analytic marginalization over the absolute magnitude of the SNIa data with a flat prior [32].

Figure 2 compares various cosmological models with some of the recent combined data from the High-Z Supernova Search Team and the Supernova Cosmology Project [1, 33, 34]. The lower figure highlights the crucial data points at the highest redshift which constrain the redshift evolution during the dark-matter dominated decelerating phase. Shown are the K-corrected magnitudes $m = M + 5\log D_L + 25$ vs. redshift.

Curves are plotted relative to an open $\Omega_{DM}, \Omega_B, \Omega_\Lambda, \Omega_\chi = 0$, $\Omega_k = 1$ cosmology.

The best-fit growing cold dark matter (GCDM) models shown on this figure correspond to either the constant U_5 or $U_5 = lH$ models. They are indistinguishable from each other on this plot. Of particular interest on Figure 2 is the fact that our best fit $\Lambda = 0$ growing cold dark matter (GCDM) models are nearly indistinguishable from the best fit Standard Λ+cold dark matter (SΛCDM) model. Thus, our model realizes the present cosmic acceleration without a cosmological constant.

This figure also illustrates an important point regarding the EOS parameter for matter in the bulk dimension. An accelerating cosmology requires that $\rho_\chi + \bar\rho$ be nearly constant. This means that no matter what the EOS for matter when it is on the brane, A constant $\rho_\chi + \bar\rho$ requires that q be small in Eqs. (24) to (29) for matter in the bulk. This is illustrated by the $q = 2$ curve of Figure 2 which is unable to reproduce the required cosmic acceleration. For the SNIa constraint alone, the best fit curve from Table 1 is for $q = 0.006$.

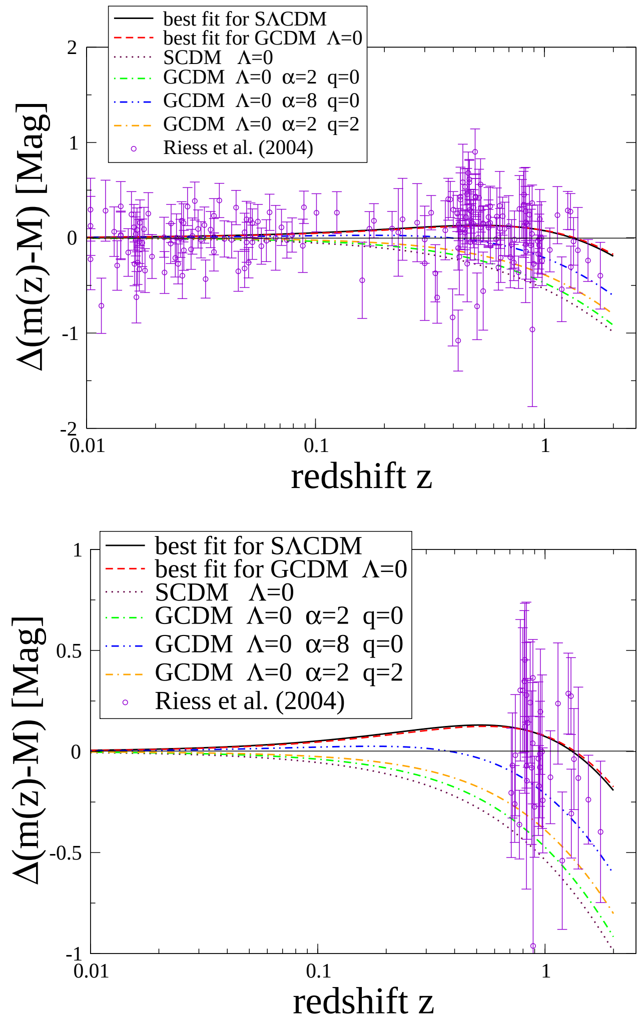

FIG. 2: Illustration of the supernovae magnitude redshift relation with and without GCDM. The upper figure shows all data set of [33]. The lower figure highlights the points with $z > 0.7$ most relevant to this paper. Note, that the best fit model is nearly indistinguishable from the SΛCDM model.

V. CMB MCMC ANALYSIS

The high-resolution measurement of the spectrum of temperature fluctuations in the cosmic microwave background (CMB) by the Wilkinson Microwave Anisotropy Probe (WMAP) has become one of the most stringent tests for cosmology. The standard procedure for obtaining cosmological parameters [35] is based upon Bayesian statistics. This approach gives the posterior probability distribution from which the optimal set of the cosmological parameters and their confidence levels can be deduced. In our analysis we followed the Markov Chain Monte Carlo (MCMC) approach [36] and explored the likelihood in an eight dimensional parameter space consisting of six WMAP standard parameters ($\Omega_b h^2$, $\Omega_c h^2$, h, $z_{\rm re}$, n_s, A_s) plus the two brane-world parameters, α and q.

Figure 3 shows best-fit theoretical CMB power spectra along with the combined data set used to constrain our models. This figure illustrates how the flow of mass-energy exchange can modify the spectrum.

There are essentially two ways in which growing cold dark matter models alter the CMB power spectrum.

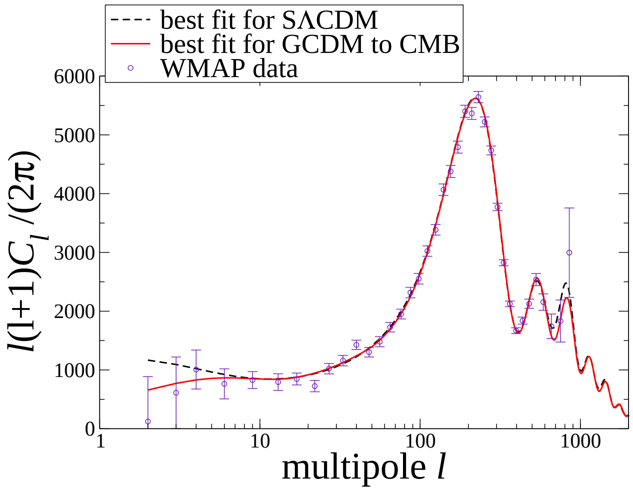

FIG. 3: CMB angular power spectrum with and without GCDM compared with observational data from WMAP. The dashed line corresponds to the best fit SΛCDM model. The solid line shows GCDM best-fit models with the cosmological constant equal to zero, and $q = 2.92$, $\alpha = 2.14$. This figure demonstrates how GCDM modifies the CMB power spectrum.

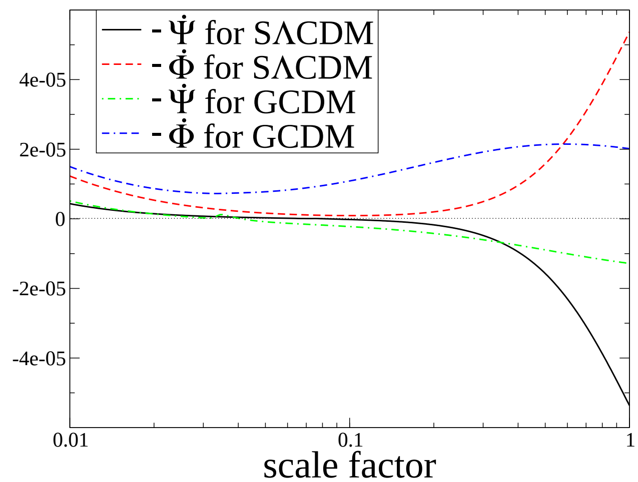

FIG. 4: Metric perturbations $\dot{\Psi}$ and $\dot{\Phi}$ for the integrated Sachs-Wolfe effect for the SΛCDM and GCDM models.

First, GCDM means that there is less dark matter at earlier times. This leads to a smaller amplitude of the third acoustic peak [37]. Second, the decay of the gravitational potential at late times is diminished due to the inflow of dark matter. This leads to a smaller late integrated Sachs-Wolfe effect (LISW) and correspondingly less power for the smallest multipoles.

The integrated Sachs-Wolf effect accounts for changes on the CMB anisotropy due the time evolution of the gravitational potentials as the photons travel from the last scattering surface to us. Writing the comoving metric perturbations as

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1+2\Phi)\gamma_{ij}dx^i dx^j] \quad (34)$$

For a photon in the absence of anisotropic stress $\Psi = \Phi$ and the ISW effect on the temperature power spectrum depends upon a simple integral over $\dot{\Phi}$ [38]. Include anisotropic stress we have:

$$\left(\frac{\delta T}{T}\right)_{ISW} = \int_{\tau_{lss}}^{\tau_0} \left[\dot{\Psi} - \dot{\Phi}\right] d\tau \quad , \quad (35)$$

where $d\tau = dt/a$ along the photon trajectory, and $\dot{\Phi} = \partial\Phi/\partial\tau$.

Figure 4 illustrates the evolution of the $\dot{\Psi}$ and $\dot{\Phi}$ potential derivatives with scale factor. In the $S\Lambda$CDM model the gravitational potentials are rapidly decaying at late times which causes enhanced power on the largest scales (smallest multipoles). In the GCDM model, however, the potentials on the largest scale are changing less. This accounts for the relative suppression of CMB power for the lowest multipoles.

For the combined SNIa and CMB data we again used the MCMC approach in a seven dimensional parameter space ($\Omega_b h^2$, h_0, z_e, n_s, A_s, α, q). It should be noted that in the GCDM cosmology of interest here we must treat Ω_b and Ω_{DM} as independent parameters in the SNIa analysis contrary to the usual cosmologies. This is because the evolution of the energy densities ρ_b and ρ_{DM} obey different functions of the cosmic scale factor a in the GCDM cosmology.

These data imply a minimum in χ^2 of 1659.8 for the GCDM model without a cosmological constant, compared to a value of 1661.7 obtained in the SΛCDM model. Note, however, that in our model there are seven parameters ($\Omega_b h^2$, h_0, z_e, n_s, A_s, α, q), while in the SΛCDM model there are only six ($\Omega_b h^2$, h_0, z_e, n_s, A_s, Ω_Λ). Hence, there is no significant improvement in the reduced χ^2 per degree of freedom as evidenced in Table 1.

Although the GCDM model explains the apparent suppression of the CMB at low multipoles, the optical depth is rather large for the optimum fit to the CMB alone ($\tau = 0.533$). This is due to a degeneracy among parameters so that a slightly better fit is obtained for the combination of a large τ offset by a smaller h and Ω_{DM}. In the combined fit with the SNIa data, however, h is better constrained so that a smaller value of $\tau = 0.133$ results. Note also, that in all of these fits a large value of $\Omega_{DR} \sim 2-3$ is obtained. This value is offset, however, by the negative dark radiation component as discussed below. The key constraint is that $\Omega_{DM} + \Omega_{DR} \approx \Omega_{DM} + \Omega_\Lambda$ as evidenced in the fifth column of Table 1.

A. Matter Power Spectrum

It is straight forward to determine the galactic matter power spectrum $P(k)$ to compare with that deduced from large-scale structure surveys [39, 40]. For the usual

power-law spectrum of primordial fluctuations

$$\delta_H^2(k) \propto k^{n-1} \quad , \tag{36}$$

a transfer function, $T(k)$ [41] is required to convert from the amplitude of the perturbation as wave number k enters the horizon, to the present-day power spectrum, $P(k)$. Transfer functions are easily computed using the code CMBFAST [42] for various sets of cosmological parameters.

An adequate approximate expression for the structure power spectrum is

$$\frac{k^3}{2\pi^2}P(k) = \left(\frac{k}{aH_0}\right)^4 T^2(k)\delta_H^2(k) \quad . \tag{37}$$

This expression is only valid in the linear regime, which in comoving wave number is up to approximately $k \lesssim 0.2\ h$ Mpc^{-1} and therefore adequate for our purposes. However, we also correct for the nonlinear evolution of the power spectrum [43].

Here it is worth noting the way in which the power spectrum is computed in our GCDM model. It is necessary to specify the distribution of the dark matter as it enters from the bulk dimension. It would be most general to suppose that there are fluctuations in the bulk mass-energy distribution just as there are in the brane. Material entering from the bulk would then appear with such fluctuations. However, we make the simplifying assumption that the dark matter and dark radiation enter with uniform distributions. After that, they are allowed to evolve as non-relativistic and relativistic particles, respectively, along with the already evolved structures on the brane.

Hence, we define the matter over density as

$$\delta = \frac{\delta\rho_{DM}^{eff} + \delta\rho_M}{\rho_{DM}^{eff} + \rho_M} \tag{38}$$

where ρ_{DM}^{eff} is the effective dark matter density as would be deduced, for example, by the gravitational potential of a galaxy cluster. For the GCDM model the effective over density is

$$\delta\rho_{DM}^{eff} \equiv \delta\rho_{DM} + \delta\rho_{DR} \tag{39}$$

and the effective gravitation mass energy density is

$$\rho_{DM}^{eff} \equiv \rho_{DM} + \rho_{DR} \quad . \tag{40}$$

As usual, a bias parameter is introduced to account for the difference between the galaxy and matter power spectrum,

$$\delta_{galaxy} = b\delta \quad . \tag{41}$$

We have made a MCMC simultaneous fit to the CMB+SNIa+$P(k)$ data. We get the best fit parameters, $q = 0.037$ and $\alpha = 8.33$. Figure 5 shows a comparison of the observed power spectrum with that of a SΛCDM

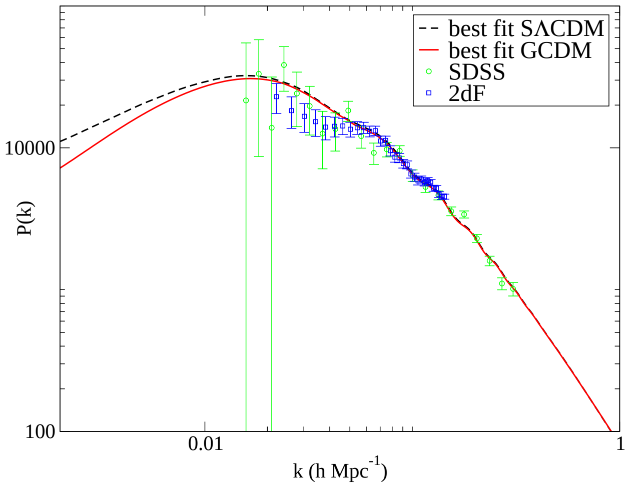

FIG. 5: Comparison of the matter power spectrum calculated in the $S\Lambda CDM$ cosmology with the $GCDM$ cosmology which best fits the CMB + SNIa + P(k) data.

model and also the GCDM model which best fits the CMB+SNIa+$P(k)$ data. The power spectrum derived in the best fit growing dark matter model is almost indistinguishable from a SΛCDM model until one gets to the very largest structures for which there is no data. The parameters associated with this model are summarized in Table 1. In these fits the bias b is a marginalized parameter. It is perhaps of note that the bias parameter deduced in this way is somewhat larger $b = 2.1$ than that deduced in the usual SΛCDM models, $b = 1.05$. This derives from the fact that the dark matter potentials are not as deep at early times, and hence, galaxy formation must be more efficient to produce the observed amplitude.

VI. CONCLUSION

We have considered models in which the apparent cosmological constant derives from the energy exchange of cold dark matter from the bulk dimension to the brane. The energy momentum tensor in this extra dimensional brane cosmology leads naturally to terms resembling a cosmological constant at the present time. If such energy exchange occurs our universe accelerates without the need to invoke a cosmological constant on the 3-brane.

We find that such GCDM exchanges are consistent with observations including the supernova magnitude-redshift relation, temperature fluctuations in the CMB, and the matter power-spectrum data. This cosmology is even slightly preferred as it fits better the suppression of the CMB power spectrum at low multipoles. We have thus demonstrated that this cosmology represents an alternative model to the SΛCDM cosmology for an observer on the 3-brane. A consistent fit to the observational constraints, however, requires that the EOS parameter for

matter in the bulk be small $q \approx 0.0$. This EOS is consistent with a need for an AdS_5 geometry in the bulk.

A peculiar feature of the present best fit models, is the fact that the true value of Ω_{DM} is much larger than in the standard cosmology, though its gravitational effect is canceled by the dark-radiation contribution. Indeed, the key constraint is that $\Omega_{DM} + \Omega_{DR} \approx \Omega_{DM} + \Omega_\Lambda$ as evidenced in the fifth column of Table 1.

One consequence of such a large and growing dark-matter contribution is the need for a somewhat large bias parameter. On the other hand, such a large dark matter content suggests new observational tests of this cosmology. For example, if the dark-matter content at the present time is much larger than in a SΛCDM model, then direct terrestrial measurements of the total density of cold dark-matter particles could indicate a much higher density than expected based upon their mass and gravitation effect. Another test of this cosmology is that there should be a suppression of the matter power spectrum on the scale of the horizon compared to a SΛCDM cosmology. Other tests of this paradigm are that a suppression of the third acoustic peak in the CMB power spectrum occurs so that better data near $l = 1000$ could help distinguish this cosmology. We also note that this cosmology produces large oscillations in the CMB polarization power spectrum so that when better polarization data are available it should help to eliminate of confirm this cosmology.

An amusing feature of this model stems from the requirement that the present dark radiation from in-flowing matter must cancel the effects of excess existing dark matter. Hence, if the flow were to cease, the universe would change to a matter-dominated $\Omega_M \approx 3$ cosmology which would begin to collapse in about a hubble time. In that sense, this is a new kind of doomsday cosmology [44].

Acknowledgments

This work has been supported in part by the Mitsubishi Foundation, the Grants-in-Aid for Scientific Research (13640313, 14540271) and for Specially Promoted Research (13002001) of the Ministry of Education, Science, Sports and Culture of Japan. K.I.'s work has been supported by a Grant-in-Aid for JSPS. Work at the University of Notre Dame supported by the U.S. Department of Energy under Nuclear Theory Grant DE-FG02-95-ER40934.

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Fit GCDM Fit SΛCDM	α	q	$\Omega_b h^2$	$\Omega_{DM}+\Omega_{DR}$ $\Omega_{DM}+\Omega_\Lambda$	Ω_{DM}	Ω_{DR} Ω_Λ	h	z_{re}	n_s	τ	b	χ_r^2
SNIa Only												
Best Fit GCDM	11.0	0.006	0.022	0.93	3.31	-2.38	0.58	-	-	-	-	1.23
Best Fit SΛCDM	-	-	0.022	0.97	0.26	0.71	0.71	-	-	-	-	1.24
CMB Only												
Best Fit GCDM	2.14	2.92	0.029	0.93	1.91	-0.98	0.64	29.1	1.18	0.533	-	1.02
Best Fit SΛCDM	-	-	0.023	0.94	0.23	0.71	0.71	14.9	0.97	0.13	-	1.01
SNIa + CMB												
Best Fit GCDM	8.45	0.023	0.023	0.95	3.14	-2.19	0.71	15.0	0.97	0.133	-	1.04
Best Fit SΛCDM	-	-	0.023	0.96	0.25	0.71	0.70	13.3	0.96	0.111	-	1.04
SNIa + CMB + P(k)												
Best Fit GCDM	8.33	0.037	0.024	0.95	3.05	-2.44	0.71	15.3	0.98	0.140	2.1	1.03
Best Fit SΛCDM	-	-	0.023	0.95	0.24	0.71	0.70	13.7	0.97	0.117	1.05	1.03

TABLE I: Parameter sets for various fits.